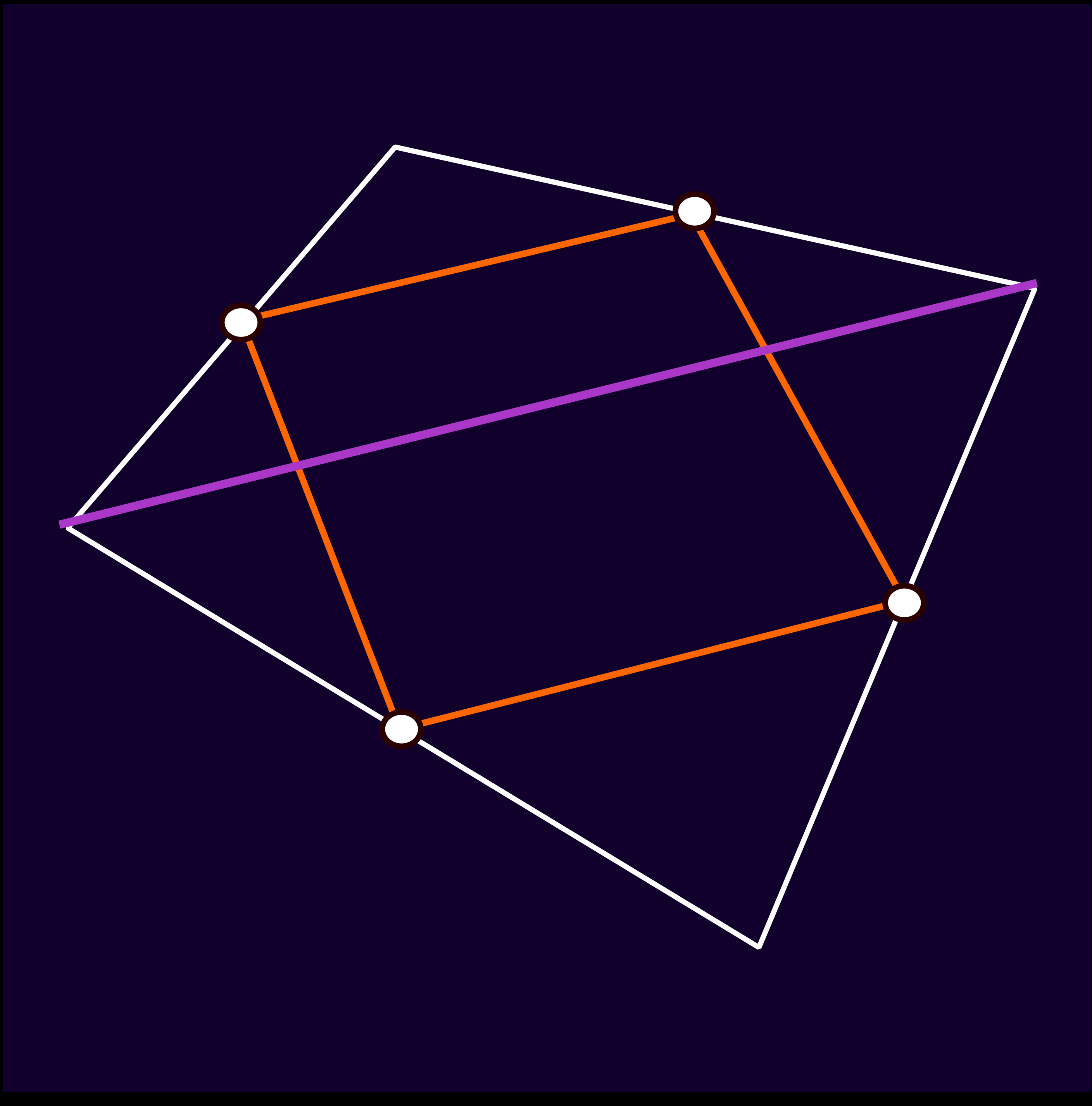
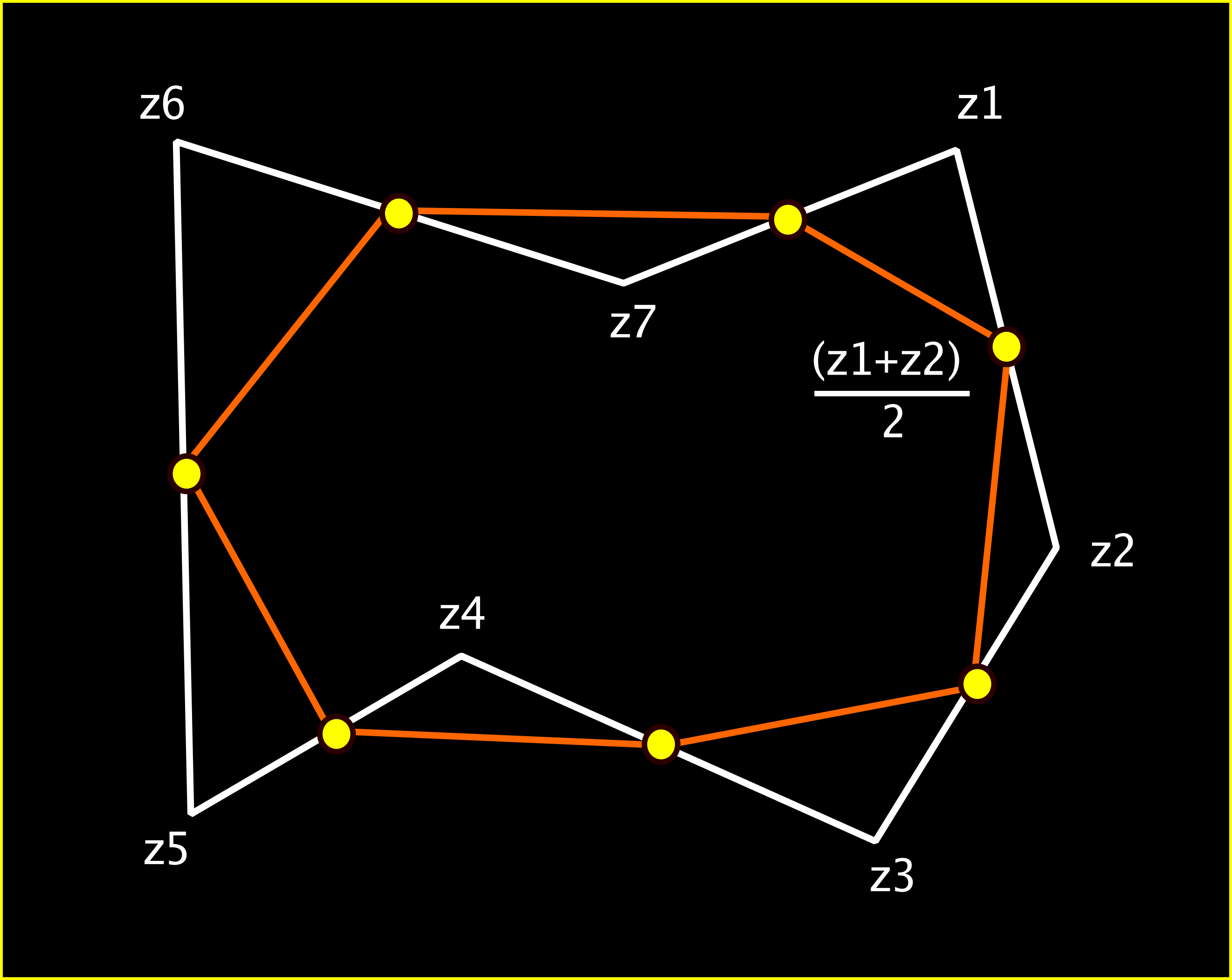
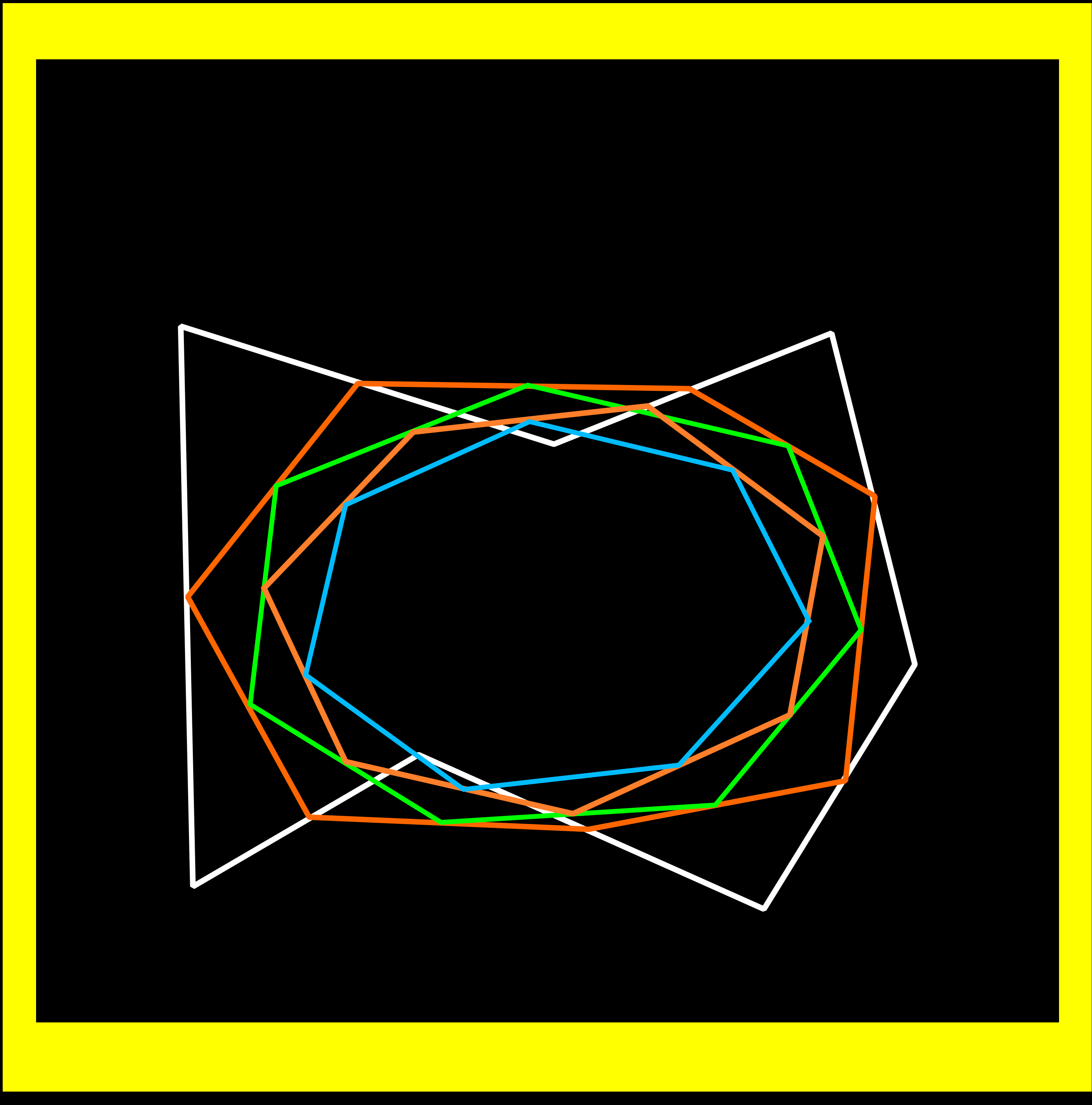


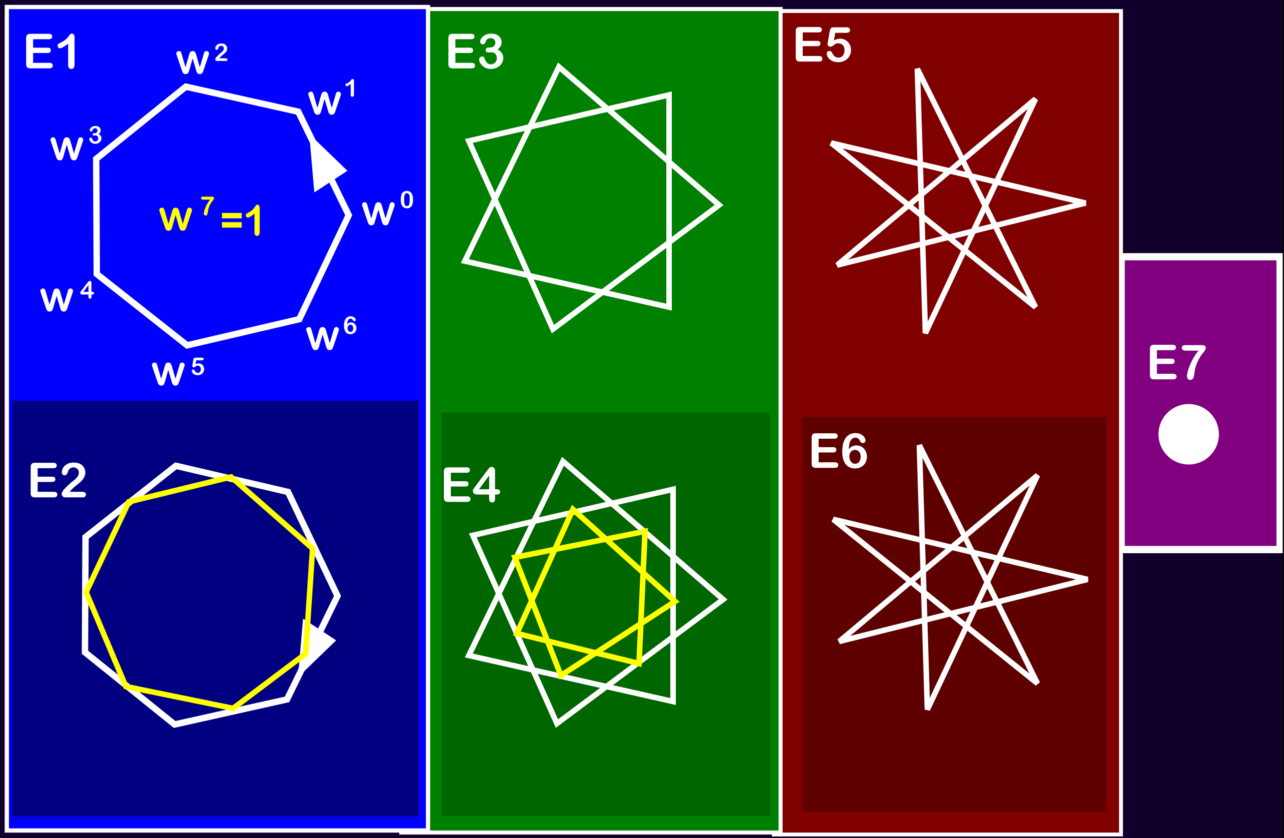
the midpoint map





Use algebra

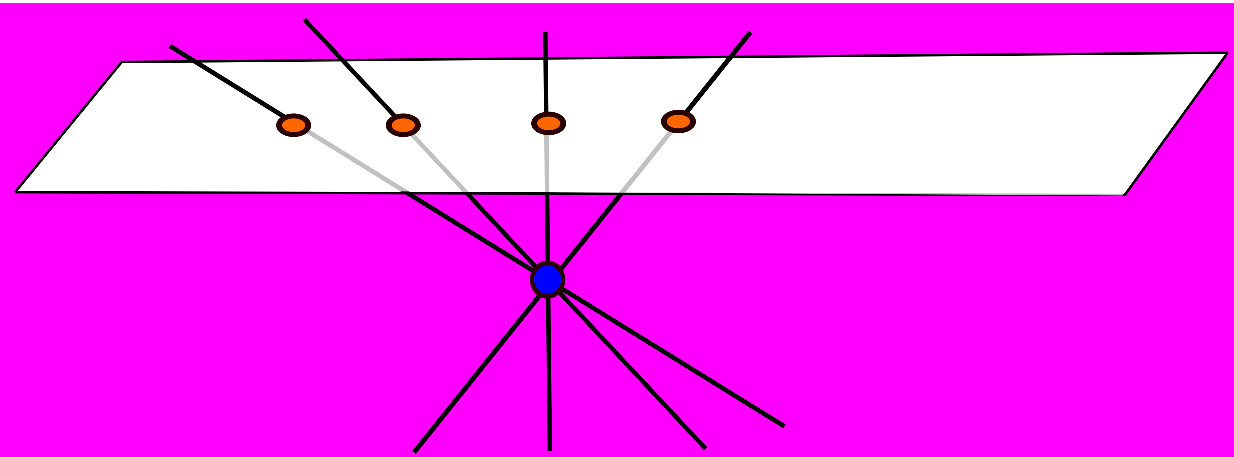
$$T(z1,\ldots,z7) = \left(\frac{z1+z2}{2}, \ldots, \frac{z7+z1}{2}\right)$$



$$T(a1\ E1 + \ldots a7\ E7) = a1\ L1\ E1 + \ldots + a7\ L7\ E7$$

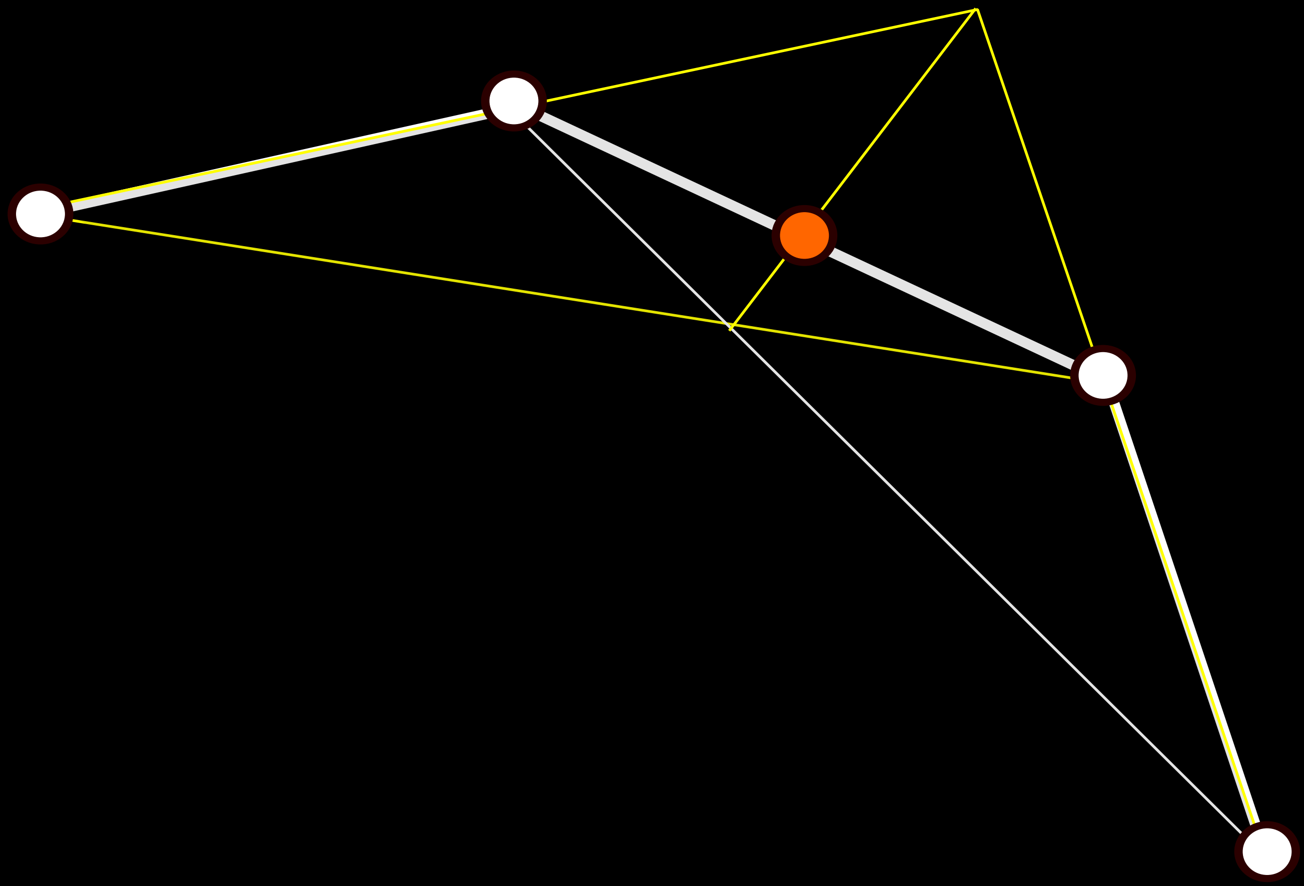
Primer on Projective Geometry:

$\mathbb{R}P^2$ = space of lines in $\mathbb{R}^3$ through the origin.

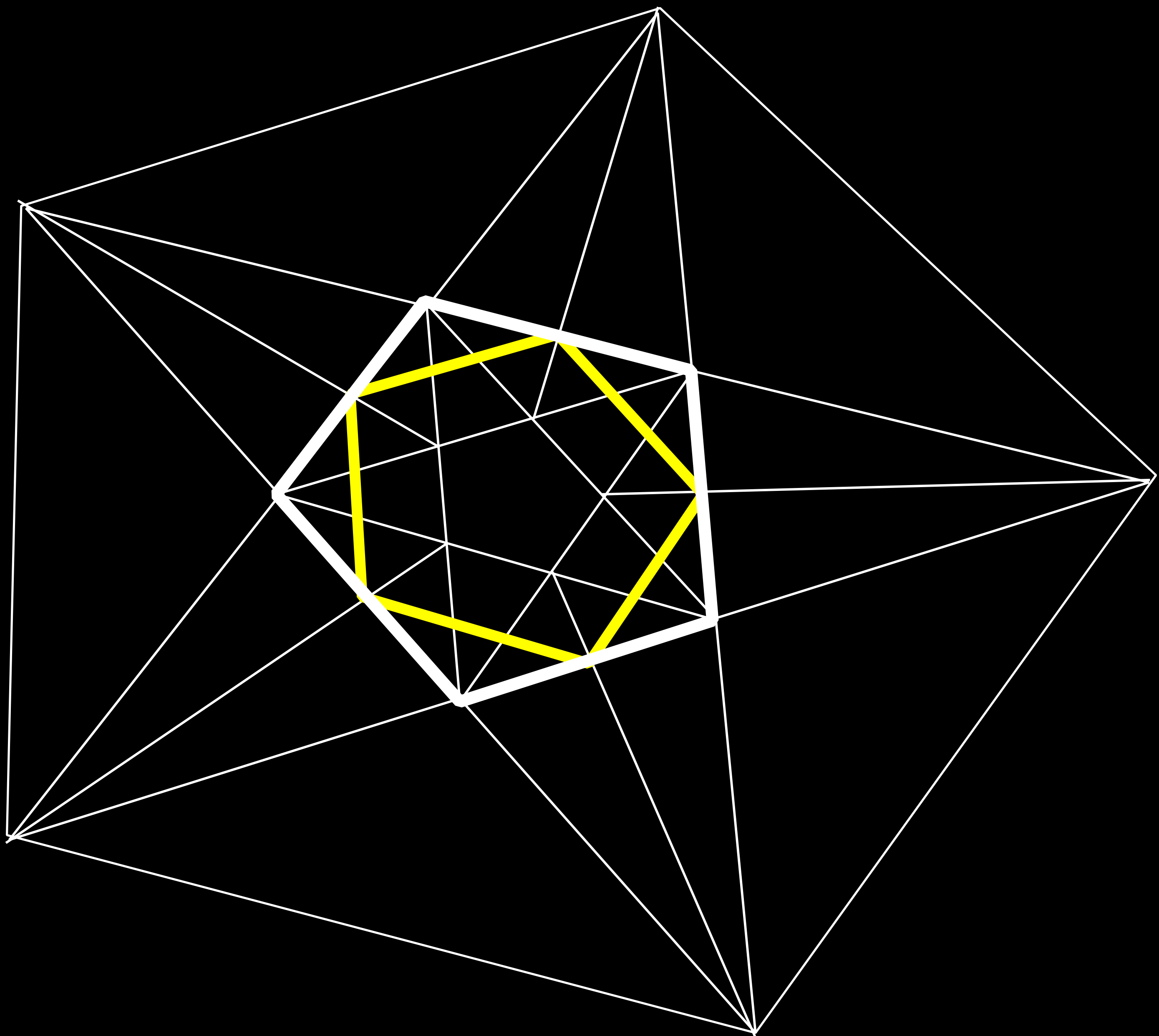


- $\mathbb{R}^2 \subset \mathbb{R}P^2$ affine patch
- Invertible linear transformations act on RP2 so as to map LINES to LINES

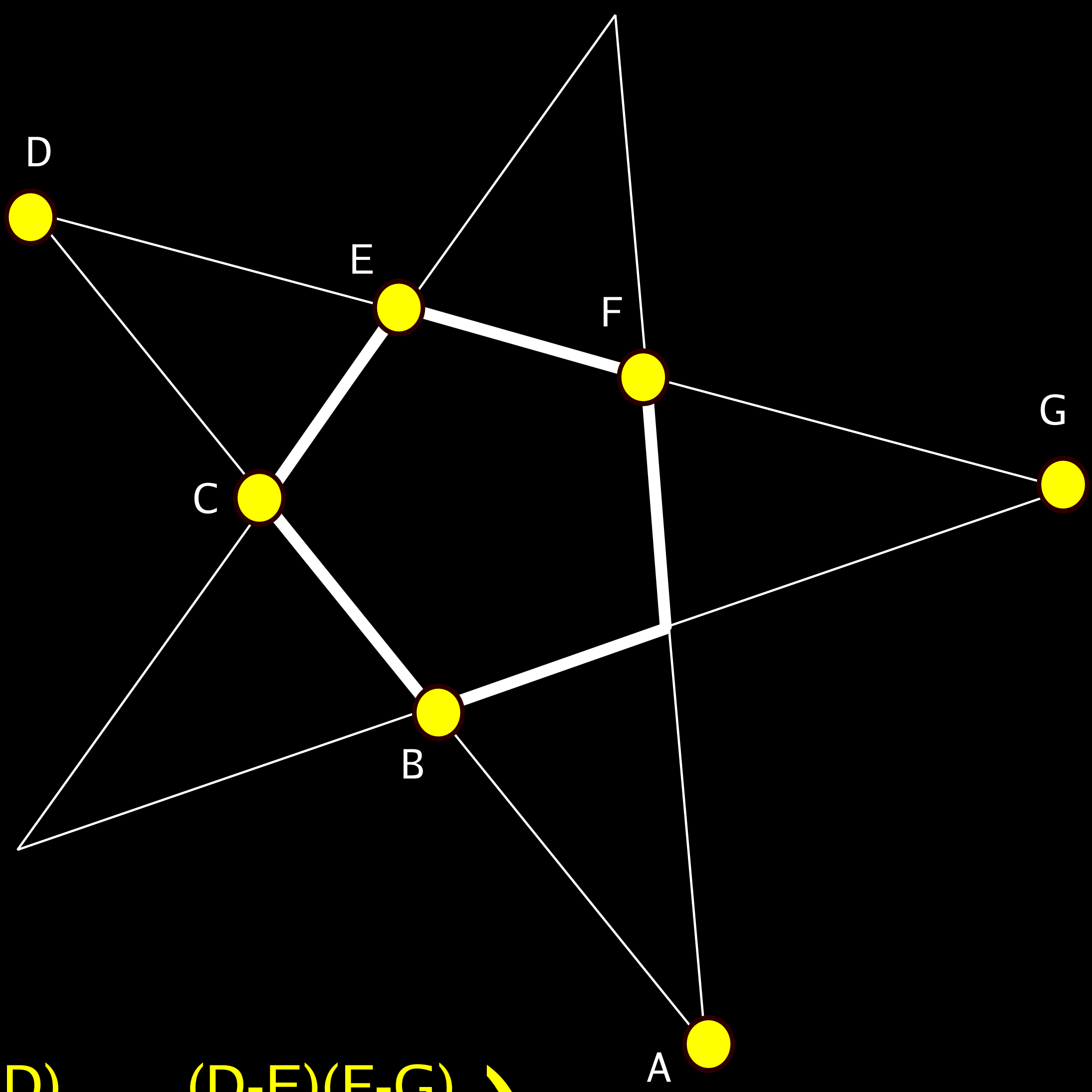
A projectively natural "midpoint".



The projective heat map:



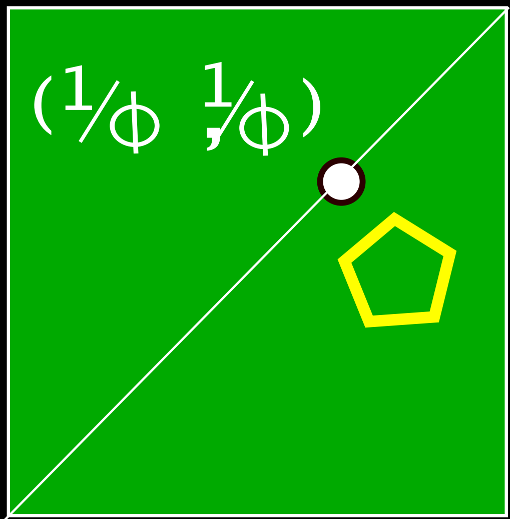
Coordinates on P_5 : first pass

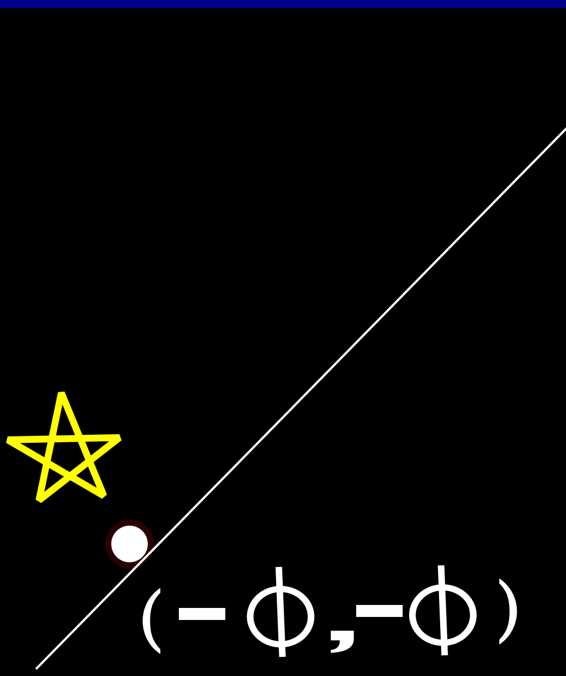


$$\left(\frac{(A-B)(C-D)}{(A-C)(B-D)} , \frac{(D-E)(F-G)}{(D-F)(E-G)} \right)$$

Theorem:

On C5 the map has the regular pentagon as a global attractor.





Proof:

The map increases the product invariant F.

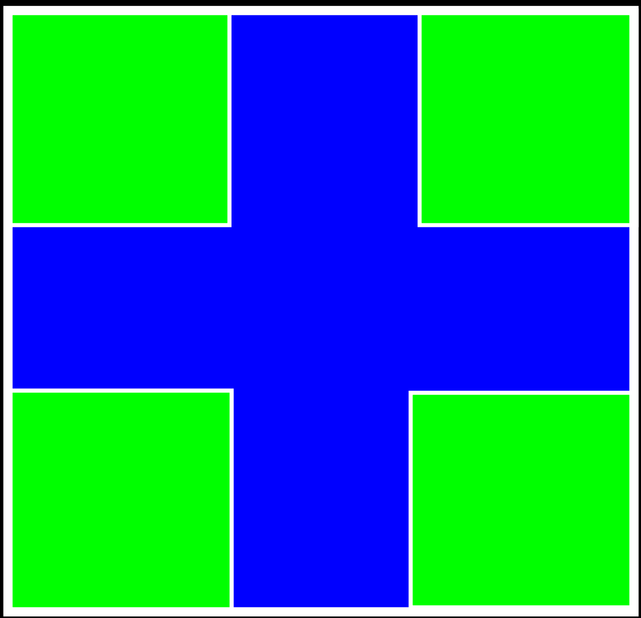
Tweak the coords:

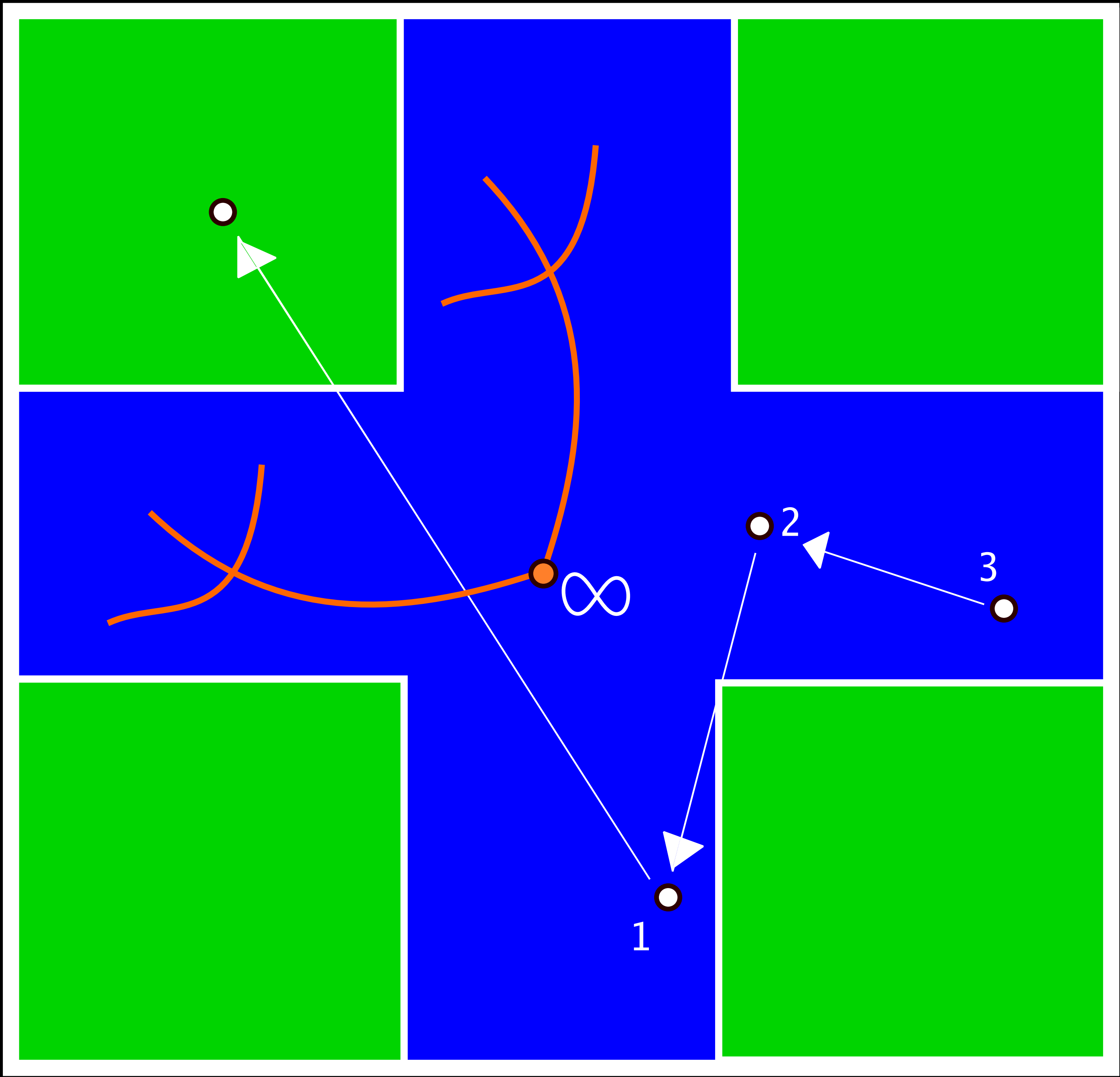
Replace (x,y) by (B(x),B(y)) so that

 = (0,0)

 =(infinity,infinity)

$$B(x) = \frac{ax+b}{cx+d}$$





2
3
∞
1

Theorem 1:

Almost every point of P5 is mapped into C5 after finitely many steps. Hence almost every point of P5 becomes asymptotically regular up to projective transformations.

Theorem 2:

WYSIWYG, except possibly for a countable union of algebraic curves.